Time: 2 hours

1. (10 points) Suppose that r indistinguishable balls are placed in n distinguishable boxes so that each distinguishable arrangement is equally likely. Find the probability that no box will be empty.

2. (10 points) A box contains 6 white balls and 9 black balls. A sample of 5 balls is drawn at random *without replacement*. Let A_3 denote the event that the ball drawn on the 3^{rd} draw is white. Let B_1 denote the event that the sample of 5 balls contains exactly 1 white ball. Find $P(A_3 \mid B_1)$.

3. Let $X \sim \text{Bernoulli}(p)$ and $Y \sim \text{Bernoulli}(q)$ be independent.

(a) (7 points) Find the distribution of $Z = X + Y - XY$.

(b) (8 points) Find the conditional distribution of $X \mid Z = 1$.

4. There are N students in the Probability class. Of them, F are female, P of them use a pencil (instead of a pen), and G of them are wearing eye glasses. A student is chosen at random from the class. Define the following events:

$$\begin{aligned} A_1 &= \{\text{the student is a female}\} \\ A_2 &= \{\text{the student uses a pencil}\} \\ A_3 &= \{\text{the student is wearing eye glasses}\} \end{aligned}$$

(a) (10 points) Let $N = 150$, $F = 90$, $P = 60$, $G = 30$. Show that it is impossible for these events to be mutually independent.

(b) (5 points) Is there an example of (N, F, P, G) where the above events are pairwise independent ?